



Viscoplastic models for high temperature applications

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Abstract

The information specific to the material deformation behavior in stress analyses of structures and manufacturing processes is contained in the material model or the constitutive equation. In the past these models had to be linear and simple to perform the stress analyses economically. Large safety (ignorance) factors made up for the simplicity. The availability of inexpensive computing power and the demands of society for reliability and safety require an improvement of stress analyses by using new and realistic constitutive equations of nonlinear, inelastic behavior of metals and their alloys. Presently used nonlinear models such as plasticity and creep theory are by far the oldest link in the stress analysis package. Test results obtained with modern mechanical testing machines and theoretical developments during the past several decades suggest that viscoplastic models using state variables are much-improved models. It is recommended that these models be carefully reviewed, further developed when needed and recommended by interdisciplinary, technology specific committees of materials, mechanics, testing and computing experts. Rather than having databases with material constants, databases containing ‘the constitutive model’ for different alloys and associated constants should be established. The analyst would call up ‘the model’, say, for annealed stainless steel and load it into the stress analysis program. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Modern society expects predictability and reliability of performance of engineering structures and their safe operation at a competitive cost. Mass produced items are usually subjected to simulated service testing to insure safety and reliability. For large structures and machinery as well as for large processes in manufacturing this approach is not possible because of the cost and the time involved. It is not possible to test a prototype power plant for 30 years before producing it!

Consequently, for one-of-a-kind machinery or manufacturing of heavy products, the performance,

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reliability and safety must be assessed early in the design stage long before the machine is built and the process started. Examples are energy generating equipment (steam and gas turbines, refineries, pressure vessels) or propulsion machinery (jet engines and rocket motors), or the manufacturing of large components. In these cases analyses of safety, reliability and performance must be made ahead of service. A key element of all of these analyses is stress analysis that yields the stresses and strains in a structure or a product under a given loading history. This analysis can then be combined with a lifetime analysis or a fracture analysis.

For the same geometry and loading of a component or product the stresses and strains will depend on the material used. Stress analyses must contain a suitable description of the material elastic and inelastic deformation behavior. This module is called the constitutive equation or material model. Viscoplasticity designates specific models used to represent the nonlinear, inelastic, rate (time)-dependent behavior of metals and their alloys.

The drive to use materials economically at low homologous temperatures requires that some inelastic deformation be allowed in the service of structures. For components operating at high homologous temperature and in manufacturing processes such as casting, rolling, extrusion and forging, the material behavior is inelastic by definition. Due to the complexity of the inelastic behavior, physically realistic inelastic analyses could not have been performed in the design office in the past. Large safety (ignorance) factors had to be used in conjunction with linear elastic analysis. The availability of large computing power at a very reasonable cost has changed the situation completely. Inelastic analysis can now be performed.

A review of presently used nonlinear, inelastic analyses reveals that very modern computer equipment is engaged to analyze materials and structures intended for future service and to analyze manufacturing processes that will produce new materials for future applications. The finite element method with plasticity and creep theory as constitutive descriptions is frequently used. These idealizations of material behavior are the oldest parts in the stress analysis package. Plasticity theory was developed around the turn of the century when Carbon steels were the major construction materials. The advent of power stations in the 1930's with lifetimes of 30 years or more required a special material model of steady behavior and creep theory was developed. Now superalloys are in use and the power plants adjust their output according to the cyclic use pattern of modern society. The material models used in design analysis are by and large the same.

The electronic revolution has also had its influence on the means of mechanical testing and data recording. This new capability makes it possible to control rates of straining or of load application and to switch instantaneously from load to displacement control. New deformation phenomena were found that classical material models couldn't predict. In some cases existing notions of inelastic deformation behavior had to be abandoned. In sum total, the results obtained with modern mechanical testing methods suggest that the old inelastic deformation idealizations such as creep and plasticity need to be improved.

If the inelastic stress analyses can be improved, the safety and reliability of machinery and processes will be increased. The development and use of realistic material models is needed and is a sure way to improve the inelastic analyses.

The computing power and the means of measuring the mechanical and thermomechanical behavior of materials are going to increase. Consequently, this article asks what kind of progress needs to be made so that the materials models are at par with the other elements of the stress analysis package. Rather than treating all material models, which would be a formidable task, this article concentrates on modeling of rate-dependent behavior of metals and their alloys in the context of viscoplasticity particularly at high homologous temperature.

2. Synopsis of the materials science approach to inelastic deformation behavior

Inelastic deformation of metals and alloys is considered a rate process and is caused by a change of the internal structure. Dislocation movements, generation and annihilation and other changes of the defect structure change in the internal make-up. The metal or alloy changes its internal structure and may become through deformation a material with different mechanical properties but with the same chemical composition of the constituents. As deformation progresses an increased force is needed for plastic flow to develop; the material hardens and exhibits an increased yield strength.

At all temperatures time-dependent deformation occurs and is important, Nix et al. (1985). Thermal softening due to diffusion processes counteracts the hardening and depends on temperature. One way to classify the influence of the diffusion is to introduce the homologous temperature defined as the ratio of operating temperature over melting temperature measured in °K. The homologous temperature will be designated by θ . The following classification is adopted from Nix et al. (1985).

1. At $\theta < 0.4$ hardening by defect structure changes and dislocation interactions predominates and the effect diffusion or recovery is small. The tangent modulus is usually positive. This temperature range is the normal operating range of structures and the cold working of metals and their alloys.
2. As the temperature increases, say $0.6 > \theta > 0.4$, diffusion competes with low temperature mechanisms with one or the other prevailing. They can also be balanced to reach a steady state deformation such as found in creep where constant strain rate can be observed. The tangent modulus is usually positive. This is the operating temperature range for power generation and propulsion machinery and consequently the deformation behavior is of considerable technological importance.
3. $\theta > 0.6$. Here the diffusion-controlled recovery is dominant and very little hardening is observed. Forming processes are performed but very few structures are operating in this temperature range. Steady creep and zero tangent modulus are observed. The transition from a solid to a fluid state is an interesting subject and is important in casting and welding simulations.

There are almost innumerable models of inelastic deformation that describe the material behavior and are used in the materials science community. They have, however, very little use in stress analysis. One of the reasons is certainly their uniaxial formulation. They describe detailed micro mechanisms, which do not fit into stress analysis where a continuum approach prevails.

3. Engineering analysis and history

A continuous transition from low-temperature deformation behavior to the melting of the alloy is the materials point of view. No sharp demarcations of various regions are considered.

In contrast, engineering analysis usually recognizes two distinct temperature regions, the low temperature regime and the creep regime. Plasticity is the model for low temperature and creep theory is used for high temperature.

As an example, the ASME Boiler and Pressure Vessel Code recognizes these regimes and defines the creep regime for ferritic materials to begin at 700°F. Below this temperature the material behavior is purported to be rate independent and plasticity is the model used. The plasticity models have their origin in experiments performed around the turn of the century when Carbon steels were the foremost construction materials. The distinct yield point of the carbon steels motivated the concept of a yield surface. In the initial formulation no growth of the yield surface and consequently no hardening was modeled. After World War II modeling of strengthening via kinematic and isotropic hardening were included.

3.1. Analyses of structures

Above 700°F creep is a significant mode of deformation and the creep theory developed in the 1930's is applied. At that time, power plants with a projected lifetime of more than 30 years were put into service. Relevant material models were needed to ensure safety and reliability of these plants and creep theory was created. The formulation is based on constant load data and the stress levels are, for ferritic steels used in power generation at least, within the quasi-elastic region of the stress-strain diagram. Analytical methods were derived for beams plates and other structures. The aim was to make sure that deformation in the creeping structure stayed below a certain value and that creep rupture would not occur.

Other alloys, the stainless steels are examples, need stress levels beyond the elastic range to obtain significant creep deformation and creep rupture in a time that is relevant for most applications. That meant that plastic strain preceded the onset of creep strain. A major challenge in modeling existed as elastic, plastic and creep strains had to be accommodated, see Odqvist (1966) for an initial account for creep theory.

During the same time-period kinematic hardening in plasticity was developed and Perzyna (1963) proposed the overstress viscoplastic model after Malvern (1951) had introduced a similar model in wave propagation analysis. The overstress concept continues to be used and developed, see the review by Krempl (1987, 1995).

A major push for the development of constitutive equations was caused by the construction of the Fast Breeder reactor. The strict requirements on safety and reliability caused a major examination of the methods of stress analysis in the creep range. At the same time the ASME Boiler and Pressure Vessel Code, Section III Nuclear Vessels had to be extended to high temperature. Code Case N 47 was created. For certain loading conditions inelastic analysis was required. Realistic constitutive equations were needed for stress analyses to meet the criteria of the Code Case N 47. Oak Ridge National Laboratory was in charge of this development and published the report by Pugh et al. (1972) entitled 'Currently Recommended Constitutive Equations for Inelastic Design Analysis of FFTF Components'. This report combined plasticity and creep theory so that inelastic design analyses could be performed.

During the preparation of this report extensive testing of stainless steel was performed and very unusual features were detected. No academician could have picked a more complex material than the stainless steels recommended by the practitioners. First they show rate dependence at room temperature and significant cyclic hardening as well as extra hardening in out-of-phase loading. Then certain versions, depending on their Carbon content, show strain aging that results in almost rate-independent behavior at certain temperatures, see Ruggles and Krempl (1989) for one example. Also the presumably 'stainless' stainless steels showed stress corrosion cracking in reactor service with drastic economic consequences for the manufacturers.

The tests on stainless steels showed that the combination of plasticity and creep equations yielded unsatisfactory modeling of certain deformation features. Included were cyclic creep and creep-plasticity interactions. It was concluded that the modeling of these phenomena was not possible within the framework of the approach proposed by Pugh et al. (1972)

Experimental evidence suggested that the separate formulation of creep theory and plasticity was the root cause. A program to develop new material models based on the 'unified' approach was started but never came to the point where new state variable theories could have been recommended for design. The breeder reactor program and with it the research and development work was terminated.

However, the advantage of unified theories had been discovered and their development continued on an individual basis. 'Unified' theories of rate-dependent deformation behavior have no separate repositories for creep and plasticity. All inelastic deformation is rate-dependent. Regular rate independence can be modeled if rate dependence is made to be small. In homogeneous motions creep and relaxation are obtained by setting the stress and strain rates, respectively, to zero. For a recent

account of the status of the state variable theories, see the articles by Chaboche; Henshall, Helling, and Miller; Krempl and others in Krausz and Krausz (1996). The common trait is the ‘unified’ approach where creep and plasticity are not separately introduced. Rather, creep is considered to be a special manifestation of rate-dependence under constant load or stress. Modeling of creep is accomplished with an appropriate formulation of the inelastic part of the flow law.

It should be remarked that the leadership of ORNL had inspired similar activities in Europe and in Japan. A sub committee of the Society of Materials Science of Japan was at work for several years, evaluated the theories, ran their own test program and issued final reports of their findings on the modeling of unified theories, see Inoue et al. (1989, 1991). These reports indicate that the development of the unified theories has not come to maturity and needs further attention.

The Japanese effort mentioned above was, like the ORNL Program, tailored to the power generation industry. For two Nickel base superalloys, B1900 + Hf and Mar-M247 NASA sponsored a program with experiments and model development, see Chan et al. (1988). Since then two Engineering Research Centers on the subject of materials modeling have been created in at the Universities in Braunschweig and in Darmstadt, Germany.

3.2. Manufacturing analyses

Initially plasticity analysis for manufacturing problems such as rolling, extrusion and forging was based on slip line fields and other analyses based on perfectly plastic or elastic perfectly plastic behavior. If elastic behavior is not included then no residual stresses can be calculated. Anisotropic yield surfaces are essential in assessing the quality of sheet metals and the earing behavior in drawing.

Recently the so-called crystal plasticity method has been applied to forming analysis. In such a method the power of finite elements is needed. The piece to be analyzed is built up from individual crystals that are allowed to interact and to re-arrange themselves according to the imposed deformation. In such an analysis the slip behavior of individual crystals is modeled using a viscoplasticity law. The viscoplasticity law is favored over a rate-independent plasticity law since it was apparently used first by Harren et al. (1989). The polycrystal plasticity approach is physically very attractive and has had many followers. It is, however, very computing intensive.

The trend in the application of constitutive equations is away from the traditional use of creep and plasticity theories. Rather viscoplastic models are used more frequently. The original Perzyna model, Perzyna (1963), was for rate-dependent plasticity, and creep and relaxation were not considered a major application. The successors to this model use it for rate dependence and for creep and relaxation. This practice is very prevalent with the ‘unified’ state variable models. The application to the modeling of forming processes also employs viscoplastic models.

Viscoplasticity is perfectly capable to model stress-strain behavior as it is affected by rate as well as creep and relaxation motions. The unified approach says that rate dependence, creep and relaxation are all interdependent and the modeling of the interdependence is sometimes very annoying, as a change in the creep behavior would also affect the relaxation behavior. This fact makes it very difficult to assess the quality of a model.

In summary, the development of inelastic constitutive equations has shown that at the present time the plasticity/creep approaches are gradually replaced by ‘unified’ methods. However, there is no consensus on specific approach and many proposals can be found.

4. Areas of application

Rate-dependent material models are found in almost any engineering application. To get an overview

Table 1
Technologies using viscoplasticity models for $\theta > 0.6$

Area	Operation	Event	Duration	Remarks
Structures	Accident analyses, Meltdown	Melting, variable temperature	Hours	Phase transformations, solid-fluid
Manufacturing	Casting	Solidification, variable temperature	Hours	Phase transformations; fluid-solid, solid-solid;
Manufact. Structures	Welding	Solidification variable, temperature	Hours	Phase transformation, fluid-solid, solid-solid
Manufacturing	Forging, Rolling, Extrusion, Superplastic forming	Variable temperature; self heating	Hours	Phase transformation

Table 2
Technologies using viscoplasticity models for $0.6 > \theta > 0.4$

Area	Operation	Event	Duration	Remarks
Manufacturing	Rolling, Extrusion, Drawing	Transient and steady conditions.	Hours	Prevent cracking, ensure quality
	Propulsion	Start-stop, steady load. Self heating, variable temperature	Hours to 10^5 hours	Ensure integrity, reliability and safety, and lifetime
Structures	Power and Process Industries	Start-stop, steady load. Variable temperature	Hours to 10^5 hours	Ensure integrity, reliability and safety, and lifetime

Table 3
Technologies using viscoplasticity models for $\theta < 0.4$

Area	Operation	Event	Duration	Remarks
Manufacturing Structures	Cold working Deep drawing, Sheet metal Penetration mechanics, Crash worthiness	Transient and steady conditions Highly transient	~ Hour < Hour	Prevent cracking, earing; ensure quality Resistance to dynamic loading important

application areas are listed according to their operating temperature. The three tables, Table 1, Table 2 and Table 3, give an overview of the technological areas and the very approximate time duration that are used in the analyses in which viscoplastic models are employed.

A. $\theta > 0.6$. The applications are mostly in manufacturing. It is desirable to model casting and solidification as well as subsequent forming operations. There are similar problems in simulating welding. Simulation of all hot forming operations is pertinent. Frequently phase transitions occur (fluid-solid) upon solidification and solid-solid transformations in forming are frequently encountered, see Thomas (1993). Structural applications are rare but can play an important role in some instances. One example is the simulation of a reactor meltdown in an accident. Then the behavior of the structure must be predicted for accident management. In all these cases the influence of variable temperature is important and must be accounted for. Variable temperature can give rise to serious thermal stresses as can deformation induced self-heating in fast operations. A summary of the application areas is given in Table 1.

B. $0.6 > \theta > 0.4$. In this temperature region hardening and recovery compete and must be accounted for. There are many manufacturing and structural applications. In the latter area a lifetime analysis complements the stress analysis. The lifetime analysis needs additional information on how damaging a certain loading will be. This subject is not considered here. The applications of viscoplasticity laws in this region are given in Table 2.

It is seen that the same problems are found in the power generation, the propulsion, the process and the electronic industries. They face the same challenges, the prevention of failure and the assurance of reliable operation. The materials used are, however, different. The other challenge is the potentially long time for which integrity has to be assured. This requirement imposes considerable emphasis on extrapolation techniques for stress analysis. The viscoplasticity law must represent the long-term creep and/or relaxation behaviors of the materials.

C. $\theta < 0.4$. This temperature range is usually considered the low temperature region of applications. However, increasingly viscoplastic laws are employed. There are for example dynamic events such as wave propagation, penetration mechanics and crash-worthiness studies that are performed with viscoplastic laws. It is generally acknowledged that dynamic deformation is rate dependent. In this temperature regime recovery is small and usually not considered. The phenomena that have to be modeled are due to hardening of the alloy or metal with rate effects.

5. Issues

5.1. Model development

The modeling of inelastic material behavior of metals and alloys is a truly interdisciplinary activity. The outcome is a material model that can be used in computer programs, especially finite element programs.

5.1.1. Regular (normal) behavior

In the forefront of model development is an understanding of the broad properties of the metal or alloy, i.e. strength and ductility changes with temperature, regions of strain aging, likelihood of precipitation reactions. It is also of interest to know the regions of temperature where normal behavior can be expected. Normal behavior includes a decrease in strength and an increase in ductility with temperature, positive loading rate sensitivity, creep and relaxation in monotonic loading. Cyclic loading involves the modeling of the Bauschinger effect and of cyclic hardening and softening. Significant creep may develop over time at stress levels that are in the quasi-elastic region of the stress-strain diagram.

Creep is of great interest as creep deformation can lead to failure of a component whereas the deformation-limited relaxation is no source of failure in monotonic loading. Repeated relaxation can lead to cracking in cyclic loading. In this case model development can follow a regular path.

5.1.2. Unusual (pathological) behavior

If strain aging (See Van den Beukel, 1975 and Nortmann and Schwink, 1997.) and precipitation reactions take place in a certain temperature-time regime unusual mechanical behavior has to be expected, see for example Rao et al. (1997). Under normal conditions an increase in loading rate leads to an increase in stress level and to a decrease of strength with increasing temperature. This is not true if strain aging is present where loading rate-insensitivity or even negative rate sensitivity and an increase in strength and a decrease in ductility with temperature are observed. If precipitates form similar unusual problems have to be expected. Modeling of these behaviors cannot be expected with normal viscoplastic laws. A special investigation and special models are needed. Metallurgists and materials scientists are very familiar with these phenomena and can help in delineating these regions of unusual behavior.

5.1.3. Modeling for normal behavior

Here the experimentalist and the mechanician are required to work together so that they can develop a model suitable for computational use. It would be ideal if a computation oriented person is involved from the beginning.

The model development starts with suitable uniaxial tests. They have to be complemented by biaxial tests since the model must be represented in tensor (matrix) form and a support by biaxial experiments is needed. Mechanicians then use theory to translate the experimental results into three-dimensional equations. Once this is done the model is ready for implementation. To be most effective team members must be willing to work across disciplinary boundaries and to assimilate the essentials of the other contributing disciplines.

Since inelastic behavior is highly nonlinear the mathematics is, by necessity and not by peer pressure, complex, especially when the three dimensional situation is addressed. Closed form solutions are usually not available and numerical integration is a necessity even for homogeneous uniaxial and multiaxial loading. Numerical experiments must be run that duplicate the real experiments. Model development requires considerable computer resources to perform the numerical experiments, evaluate the results and to deliver the data. Ordinary differential equation solvers such as LSODA, see Hindmarsh (1983), are needed to integrate the model when it is applied to the test conditions, which involve uniaxial tests, as well as biaxial tests.

5.1.4. Consensus building

In the linear case it is comparatively simple for a group of people to select the best model when the experimental results are compared with the model predictions. In fact a linear regression analysis can make this comparison objective. The situation is much more complex for viscoplasticity. Not only is the model nonlinear, but it can be applied to monotonic and cyclic loading, creep and relaxation and each response is nonlinear. It is necessary to perform an optimization analysis to decide on the goodness of the model. Unlike the linear regression analysis an optimization program is not readily available. Judgements from different experts are subjective but are, at the moment at least, the most reliable methods. No standard exists relative to which particular results could be compared to.

It is necessary that there be a consensus what a model can accomplish and what are normal or acceptable deviations. A consensus has to be built through suitable committees. The members of such a committee will spread the word to their colleagues at their place of employment. Such a consensus building has been accomplished in the ASME Boiler and Pressure Vessel Code and in the Japanese committees that published Inoue et al. (1989, 1991).

5.1.5. Model types and acceptance

At the present time many different models are in use. They differ in terms of their form, algebraic vs. incremental, and with respect to the variables in use, e.g. stress, strain, strain rate or stress, strain rate and state variables. In the latter category the number of state variables differ from author to author and range from one to three in applications.

There is also a difference in the goals that model developers have. Some authors provide a curve fit to the experimental data that are aimed at reproducing only the observed data. An example is finding a mathematical expression between stress, strain rate and time (or creep strain) to reproduce a set of creep curves. This method has been called ‘*Creep Empiricism*,’ see Kennedy (1963). Such models are not suitable for use in computer analysis as they do not properly represent the behavior of the metal or alloy in other than constant stress (load) condition.

On the other side of the spectrum are the state variable theories that do model the behavior in more than a single type of homogeneous deformation. For a state variable theory to describe a certain behavior it is necessary to specialize the theory for that test, e.g. total strain rate is zero for relaxation, to supply the initial values and to integrate the state variable model. The result is a relaxation curve. The procedure has to be repeated for another test, say a stress-controlled monotonic loading.

At the present time viscoplasticity models are frequently employed to model only specific behaviors such as loading rate sensitivity, but not creep and relaxation, creep but not relaxation and not cyclic loading. State variable models can reproduce all these phenomena and there is, in principle, no reason to restrict the model to certain histories. This general capability can turn into a liability. A state variable model may not be as accurately matching the creep data as ‘*Creep Empiricism*’. However, state variable models can be applied to monotonic stress-controlled loading and many other conditions with frequently very acceptable results. The model obtained with the ‘*Creep Empiricism*’ is only capable of reproducing this behavior.

Given that the state variable models can in principle be applied to any condition of loading, it would appear to be feasible to develop a state variable model for a particular alloy, say of a stainless steel or an Aluminum alloy at small strain. Rather than taking the elastic modulus and Poisson’s Ratio from a database, the state variable model with the material constants would be selected from a database of material models. Then the stress-strain behavior under a variety of loading could be predicted. An even more ambitious scenario would be a database of models for a certain alloy at small and large strains with allowance of different heat treatments!

It appears that the available incremental computational power has eliminated the need for using algebraic models for simplicity. Unlike incremental models that can be adapted to any kind of loading, algebraic models are only good for the condition of the test program.

6. Recommendations

The premises of the recommendations are the following observations:

- Inexpensive computing power is ubiquitous.
- The material idealizations used in the majority of inelastic stress analyses are more than fifty years old.
- The observations leading to these idealizations were obtained with testing machines that are completely outdated compared with present capabilities.
- The computer revolution has increased the capabilities of determining the metal (alloy) properties. Rate effects and cyclic loading can be accurately controlled and measured.

These facts suggest the following actions with the goal of making the constitutive equation commensurate with the other elements of the stress analysis package.

- Classify available viscoplastic models according to their capabilities to reproduce certain phenomena and their complexity.
- Make a survey and an assessment of the viscoplastic models used in technology. The technological areas listed in Table 1 could serve as a starting point. Assess the models used relative to the capabilities of the presently available models. Give a recommendation how the models can be adapted to serve the needs of different technological areas.
- Evaluate the models regarding their capability and complexity. Recommend certain models for adaptation with or without further development.
- Establish consensus. Entice computation, modeling, testing and materials specialists to cooperate in the assessment and the development of models.
- The goal is to establish ‘the constitutive equation’ that completely represents the inelastic behavior of an alloy, say 316 stainless steel or Aluminum for small strain. A more ambitious project would be to develop a model for large deformation. Assemble these models and the model specific material constants in suitable databases to facilitate their use in academia, government and industry.

Acknowledgements

The support of the Department of Energy, Basic Energy Sciences, Grant DE-FG02-96ER14603 is gratefully acknowledged.

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